DYNAMICS OF SURFACE BUBBLE EXPANSION INVOLVING THE THERMOPHYSICAL PROPERTIES OF THE HEATER

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The results are presented of a theoretical study, according to the asymptotic microlayer model, of the expansion of surface bubbles at values of the Jakob number above 30. The nonsteady influence of the thermophysical properties of the heating surface is accounted for in the solution obtained by the method of quasisteady thermal approximation.

As the heat load increases during bubble boiling, the fraction of heat carried by a vapor bubble away from the heating surface increases and becomes dominant [1]. This experimentally established fact underlines the need for and urgency of studying the macrocharacteristics of the boiling process and, particularly, the anomalies of the rate of expansion of surface bubbles. Earlier models of expansion of vapor bubbles at the surface of a heat-transmitting wall do not take into account the influence of the heater material [2-9]. At the same time, the results of studies of nitrogen boiling, for instance, at a heating surface made of various different metals indicate a strong dependence of the bubble expansion dynamics on the thermophysical properties of the wall material [10].

The object of this study is to derive an expression for the expansion dynamics of a vapor bubble which will take into account the thermophysical properties of the heat-transmitting wall.

The existence of a thin liquid boundary layer under an expanding bubble has been demonstrated by both indirect thermometric and direct optical methods [11, 12]. The volume of vapor entering a bubble contains the product of liquid evaporation from the hemispherical surface as well as from the microlayer surface. During the asymptotic stage of hemispherical bubble expansion, when the bubble radius is much larger than the thickness of the thermal boundary layer, evaporation from the curved surface remains negligible so that [13]

$$V_{\mathrm{ML}}=\frac{2\pi R^3}{3}=\frac{\rho_2}{\rho''}\left\{\int\limits_0^{R_{\mathrm{d}}}\delta_0(t)\,2\pi r dr+\int\limits_{R_{\mathrm{d}}}^{R}(\delta_0(t)-\delta(t))\,2\pi r dr\right\}. \tag{1}$$

Integration of expression (1) yields the law according to which a vapor bubble expands when the microlayer evaporation is a given function of time.

The dynamics of microlayer evaporation at a solid surface $\delta(t)$ can be determined from the solution to the one-dimensional problems of heat conduction, inasmuch as $\delta \ll R$. A theoretical calculation is in the general case not possible, owing to the nonlinearity of the heat problem. Neither can the problem be solved by choosing an approximation for $\delta(t)$ with the use of series and special functions [14].

Difficulties with obtaining an analytical expression for $\delta(t)$ have stimulated a search for physically valid simplifications. Foremost among them are the assumptions of a uniform initial distribution and of a negligible heat capacity of the microlayer. The latter implies that the temperature field of the liquid layer can be regarded as being quasisteady, entirely plausible in the case of a thin microlayer. With these assumptions, then, evaporation of a liquid film at the surface of a massive body is described by the differential equation of energy transfer in the wall

$$\frac{\partial T_1(x,\ t)}{\partial t}=a_1\frac{\partial^2 T_1(x,\ t)}{\partial x^2},\quad 0\leqslant x\leqslant\infty,\quad t>0; \tag{2}$$

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with the boundary conditions

$$\left.\frac{\partial T_1(x,\ t)}{\partial x}\right|_{x\to\infty} = 0, \tag{3}$$

$$\lambda_1 \frac{\partial T_1(0,\ t)}{\partial x} = \lambda_2 \frac{T_1(0,\ t) - T_2(\delta(t),\ t)}{\delta(t)}, \tag{4}$$

$$\frac{\partial T_1(0,\ t)}{\partial x} = -\frac{\rho_2 L}{\lambda_1}\frac{d\delta}{dt}, \tag{5}$$

$$T_2(\delta(t),\ t) = T_s = \text{const} \tag{6}$$

and the initial conditions

$$T_1(x,\ 0) = T_{10}(x), \tag{7}$$

$$\delta(0) = -\delta_0. \tag{8}$$

Although the equation of heat conduction for the liquid film has been eliminated here, the nonlinearity of boundary condition (5) still makes it difficult to solve the problem.

Several authors tried to solve the problem formulated in this manner. Application of the approximate integral method [15] has not yielded a $\delta(t)$ relation in explicit form [16]. For this reason probably, the expansion model for a spherical bubble at the heating surface in [17] was based, in addition to these assumptions, on a third assumption of a constant microlayer thickness: $\delta(t)$ = const. In the derivation of the equation for the evaporation rate of a liquid film [17] was used the corresponding approximation to the exact solution describing the temperature field of a semibounded solid body with a uniform initial distribution and with heat transfer from the surface to the medium at zero temperature [18]. The resulting equation of the vapor bubble dynamics taking into account the thermophysical properties of the heater material is thus of a semiempirical nature, inasmuch as it includes two correlating constants.

Essentially the same assumptions were used in a later study [19]. Disregarding the formation of a dry zone, i.e., letting $R_d = 0$, one calculated there the volume of vapor evaporated from the microlayer by a numerical method. As a result, the following empirical expression for the expansion of a hemispherical bubble was fitted

$$R = \frac{2\,\mathrm{Ja}\sqrt{a_2 t}}{C_2\sqrt{\mathrm{Pr}} + \sqrt{\dfrac{\pi}{k_\varepsilon^2}}}, \tag{9}$$

with constant C_2 taken from the equation describing the initial thickness $\delta_0 = C_2\sqrt{\nu_2 t_2}$ of the liquid film forming under the bubble and determinable either analytically [20] or experimentally [21].

In another study [22] the problem (2)-(8) was solved analytically by the method of quasisteady thermal approximation. The resulting theoretical relations for the dynamics of microlayer evaporation, fluctuations of the surface temperature, total evaporation time, and others were compared with the results of numerical solution on a digital computer [23]. Analytical and numerical data almost coincided, within the accuracy of graphs used for computation.

The main advantage of the solution in [22] is the explicit form of the equation of dynamics. It thus can be analyzed and simplified, as necessary, for evaluating the integral in expression (1). Accordingly, the evaporation rate of the film is

$$\frac{d\delta}{dt} = -\frac{k_\lambda^2 a_1}{4\delta^2}\left[\left(\frac{4k_\varepsilon^2}{M} - 3\right)\delta - 3\delta_0 + \sqrt{9(\delta+\delta_0)^2 - 24\frac{k_\varepsilon^2}{M}\delta(\delta+\delta_0)}\right]. \tag{10}$$

We now write the integral of expression (10) as

$$\left(4 - \frac{3M}{k_\varepsilon^2}\right)\frac{\delta^2 - \delta_0^2}{2} - \frac{3M}{k_\varepsilon^2}\delta_0(\delta+\delta_0) - \int_{\delta_0}^{\delta}\sqrt{\frac{3M}{k_\varepsilon^2}\left[\frac{3M}{k_\varepsilon^2}(\delta+\delta_0)^2 - 8\delta(\delta+\delta_0)\right]}\,d\delta = -\frac{4a^2 t}{M}.. \tag{11}$$

This is an equation which describes, in a general form, the law of motion of the film boundary during evaporation. The integral evidently involves elementary functions, but is rather intricate in nature and divides into three ranges determined by the value of the complex

$$\beta = \tilde{m}/k_\lambda = k_\varepsilon^2/M.$$

An analysis of the integrand function (11) reveals that the second term under the radical sign contributes little to the total value (it vanishes at the endpoints of the 0, $-\delta$ interval) and can be disregarded. With this simplification, we have the following law of film evaporation dynamics:

$$3 - \frac{2k_\varepsilon^2}{M}\,\delta^2 - 6\delta\delta_0 + \left(3 + \frac{2k_\varepsilon^2}{M}\right)\delta_0^2 = \frac{4k_\varepsilon^2}{M^2}\,a_2 t. \tag{12}$$

The largest error of this analytical solution (12) does not exceed 20% of the numerical solution in [23] for small values of β. For large values of β the results according to Eq. (12) are almost identical to those obtained on the digital computer. Integration of Eq. (12), which is necessary for the subsequent derivation of the equation of bubble expansion, will furthermore certainly reduce the error in the final expression for the dynamics of a surface bubble.

Equation (12) readily yields an expression for the total evaporation time t_e so that the radius of the dry zone can be determined. Letting $\delta = 0$, we have

$$t_e = \frac{\left(3 + \frac{2}{M}k_\varepsilon^2\right) M^2\delta_0^2}{4k_\varepsilon^2 a_2} \tag{13}$$

or

$$\mathrm{Fo}_e' = \frac{a_1 t_e}{\delta_0^2} = \frac{3 + \frac{2k_\varepsilon^2}{M}}{4\tilde{m}^2}. \tag{13'}$$

The asymptotes of this equation are

for $k_\varepsilon^2 \to 0$ $(k_\lambda \to \infty)$

$$\mathrm{Fo}_e' = \frac{3}{4\tilde{m}^2}; \tag{14}$$

for $k_\varepsilon^2 \to \infty$ $(k_\lambda \to 0)$

$$\mathrm{Fo}_e' = \frac{1}{2\tilde{m}k_\lambda}. \tag{15}$$

The solution to the equation of microlayer evaporation dynamics (12) yields the instantaneous film thickness

$$\delta = \frac{3\delta_0 M}{(3M - 2k_\varepsilon^2)} - \frac{2\sqrt{k_\varepsilon^2}\,\sqrt{k_\varepsilon^2 M\delta_0^2 + a_2(3M - 2k_\varepsilon^2)(t - t_2)}}{\sqrt{M}\,(3M - 2k_\varepsilon^2)}. \tag{16}$$

Integration of expression (1) from 0 to R, with the use of expression (16) but the effect of the dry zone disregarded, yields the bubble residue as a function of time

$$R = \frac{2\mathrm{Ja}}{C_2\sqrt{\mathrm{Pr}} + \sqrt{\frac{3}{k_\varepsilon^2} - \frac{2}{M}}}\sqrt{a_2 t}. \tag{17}$$

A comparison between this expression and the empirically fitted approximation (9) on the assumption that $M \to \infty$, equivalent to $\delta(t) = \mathrm{const}$, reveals that they differ only in their constants; here we have $\sqrt{3}$ instead of $\sqrt{\pi}$.

Transforming Eq. (16) by the method outlined in [13] and letting $\delta = 0$, we obtain for the radius of the dry zone R_d

$$R_d = 2R\left[\left(\frac{3M}{2k_\varepsilon^2} + 2\right)C_2^2 \Pr M + 4\right]^{-\frac{1}{2}}. \tag{18}$$

As the general activity coefficient increases, the radius of the dry zone obviously increases toward a limiting magnitude. Increasing the phase transition number M and $C_2\sqrt{\Pr}$ will decrease R_d. Inasmuch as $C_2\sqrt{\Pr}$ and M increase with decreasing pressure, radius R_d will become maximum under a nearly atmospheric pressure. Calculations according to expression (18) agree closely with the experimental data in [24] on the radius of the dry zone.

Taking the dry zone into account produces a more general expression for the dynamics of surface bubbles at intermediate values of the thermal activity coefficient, namely

$$R = \frac{2\,\mathrm{Ja}\,C_2\sqrt{\Pr}\,\sqrt{a_2 t}}{C_2^2 \Pr + \dfrac{4k_\varepsilon^2}{M(3M+2k_\varepsilon^2)} + \dfrac{3\sqrt{M}}{\sqrt{k_\varepsilon^2}\sqrt{3M+2k_\varepsilon^2}}\sqrt{C_2^2 \Pr + \dfrac{4k_\varepsilon^2}{M(3M+2k_\varepsilon^2)}}}. \tag{19}$$

From this equation can be obtained, as special cases, the asymptotes

for $k_\varepsilon^2 \to \infty$

$$R = \frac{2\,\mathrm{Ja}\,C_2\sqrt{\Pr}}{C_2^2 \Pr + \dfrac{2}{M}}\sqrt{a_2 t}; \tag{20}$$

for $k_\varepsilon^2 \to 0$

$$R = \frac{2\,\mathrm{Ja}\sqrt{k_\varepsilon^2}}{\sqrt{3}}\sqrt{a_2 t}. \tag{21}$$

We note that at high values of the phase transition number M expression (20) is completely identical to, and expression (21) differs only in the constant from the corresponding extremal solutions obtained in [21], for instance, through integral estimates.

The theoretical equation (19) describing the dynamics of a surface bubble was also compared, with regard to its basic features, with the experimental data in [25] for water and several organic liquids. This comparison indicates that $C_2 = 1.27$ yields the closest agreement with experimental data in the case of water with the Jakob number within the 30-800 range, i.e., that no empirical correlating coefficients are needed then. For organic liquids, which are more viscous, the smallest theoretically possible value $C_2 = 0.866$ is preferable.

An analysis of Eq. (19) indicates, furthermore, that the surface material influences the rate of bubble expansion most at low values of the complex $C_2 \Pr^{1/2}$, i.e., under nearly atmospheric pressures. As the pressure drops, the thermophysical properties of the heater material influence the dynamics of bubble expansion and thus also the heat transfer during boiling less and less.

We note that the lower limit of applicability of the said microlayer model is analytically determined by the condition that $\mathrm{Ja} > 30$ [20]. Determining the upper limit of the Jakob number at which integral effects become so significant as to render Eq. (19) inapplicable requires additional experiments according to the procedure outlined in [16, 25].

NOTATION

a	is the thermal diffusivity;
C_p	is the heat capacity;
ν	is the kinematic viscosity;
T	is the temperature;
L	is the heat of evaporation;
ρ	is the density;

t	is the time;
x, r	are the space coordinates;
δ	is the microlayer thickness;
R	is the bubble radius;
V	is the volume;
C_2	is the constant;
t_2	is the time of bubble expansion to a given present radius;
$Ja = Cp_2\rho_2(T_{10} - T_s)/\rho''L$	is the Jakob number;
$Pr = \nu_2/a_2$	is the Prandtl number;
$k_\varepsilon^2 = \lambda_1 Cp_1\rho_1/\lambda_2 Cp_2\rho_2$	is the thermal activity coefficient;
$k_\lambda = \lambda_2/\lambda_1$	is the criterion characterizing the thermal conductivity of the liquid referred to that of the wall material;
$M = L/Cp_2(T_{10} - T_s)$	is the phase transition number;
$\tilde{m} = \rho_1 Cp_1(T_{10} - T_s)/\rho_2 L$,	
$\beta = m/k\lambda$	are the evaporation parameters.

Subscripts

1	refers to the wall surface;
2	refers to the liquid;
0	refers to the initial state ($t = 0$);
s	refers to parameters of the saturation curve;
''	refers to vapor;
e	refers to the end of evaporation;
ML	refers to the microlayer;
d	refers to the dry zone.

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INVESTIGATION OF THE COOLANT EDGE WETTING ANGLE FOR MESH HEAT PIPE WICKS

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Results are presented of an experimental investigation of the edge wetting angle of Freon-22, ethane, and ammonia for mesh wicks in the 20-115, 6-115, and 20-60°C temperature ranges, respectively.

The effective units produced recently for heat transmission, namely, heat pipes [1, 2], are being utilized all the more extensively in engineering, including even in the 150-273°K temperature range, because of a number of their inherent properties: the transmission of considerable heat fluxes at low temperature drops, the capability of producing isothermal conditions in relatively large areas, the possibility of transforming a heat load, the negligible weight, the absence of moving parts and the requirement of pumps, and the total autonomy.

A heat pipe transmits heat flux from the heating to the cooling zone by means of evaporation and condensation of the intermediate heat carrier. Reverse motion of the liquid from the condenser section to the heat supply zone is accomplished because of the capillary forces by means of a capillary-porous structure (wick) located on the inner heat pipe surface.

One of the most widespread wick materials for heat pipes is a metal mesh. In order to develop reliable methods of computing and designing heat pipes with mesh wicks, more complete knowledge of the capillary properties of the metal mesh structures is needed. The wick structure in any case of heat pipe utilization should assure delivery of the surface being cooled by the heat carrier. The physical processes occurring in heat pipes impose a number of constraints on their heat-transmitting capability, which are associated, in particular, with the greatest achievable capillary head, which is determined by the Laplace equation for a structure with cylindrical pores

$$\Delta p_c = 2\sigma\cos\theta\left(\frac{1}{R_1}-\frac{1}{R_2}\right). \tag{1}$$

It follows from (1) that the edge wetting angle θ substantially influences the quantity Δp_c.

A number of papers devoted to the investigation of the hydrodynamic and structural characteristics of different classes of porous materials used as heat pipe wicks has recently been published in the literature [3-6].

However, the angle θ is often assumed to be 0° [7-8] in computations of the transport properties of wicks although wetting of the wick structure is far from ideal in the majority of cases, and different working liquids wet the capillary-porous structure with a specific edge wetting angle θ in every case. The edge wetting angle is an important characteristic of the metal–liquid combination which can be used as a wick and heat carrier of a specific heat pipe. Because of the complexity of the analytical computation of values of θ, experimental values of the edge angle are of practical value.

Information about the edge angles is quite scarce in the literature and practically absent for liquids in the temperature range of cryogenic and low-temperature heat pipe operation [9-11].

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